## Exercise 36

Suppose that an object moving in direction $\mathbf{i}+\mathbf{j}$ is acted on by a force given by the vector $2\mathbf{i}+\mathbf{j}$. Express this force as the sum of a force in the direction of motion and a force perpendicular to the direction of motion.

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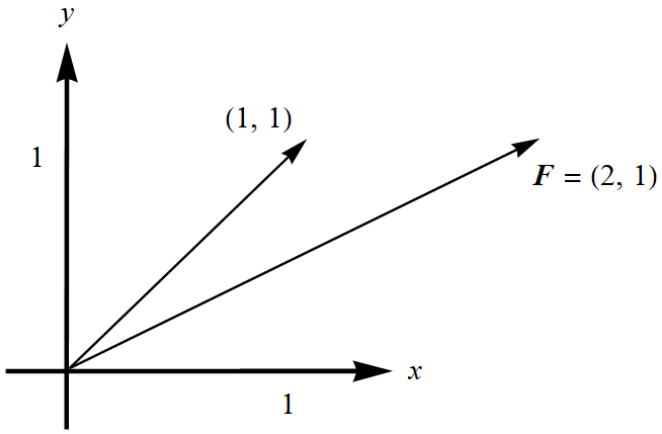
**Solution**




The aim is to decompose the force vector $\mathbf{F}$ into components parallel and perpendicular to the direction of motion.

$$\mathbf{F} = \mathbf{F}_{\parallel} + \mathbf{F}_{\perp}$$

$\mathbf{F}_{\parallel}$ will be found first. Start by taking the dot product of $\mathbf{F}$ with a unit vector in the direction of motion.

$$(2,1) \cdot \frac{(1,1)}{\sqrt{1^2+1^2}} = \frac{(2,1)\cdot(1,1)}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

This represents the component of $\mathbf{F}$ parallel to the motion. Multiply it by the unit vector in this direction to get $\mathbf{F}_{\parallel}$.

$$\mathbf{F}_{\parallel} = \frac{3}{\sqrt{2}}\frac{(1,1)}{\sqrt{1^2+1^2}} = \frac{3}{2}(1,1)$$

To get $\mathbf{F}_{\perp}$, subtract $\mathbf{F}_{\parallel}$ from $\mathbf{F}$.

$$\begin{aligned}
\mathbf{F}_{\perp} &= \mathbf{F} - \mathbf{F}_{\parallel} \\
&= (2,1) - \frac{3}{2}(1,1) \\
&= (2,1) - \left(\frac{3}{2}, \frac{3}{2}\right) \\
&= \left(\frac{1}{2}, -\frac{1}{2}\right) \\
&= \frac{1}{2}(1,-1)
\end{aligned}$$

Therefore,

$$\mathbf{F} = \frac{3}{2}(1,1) + \frac{1}{2}(1,-1).$$